UT-KOMABA 00-09

Three point functions and the effective lagrangian for the chial primary fields in $D = 4$ supergravity on $AdS_2 \times S^2$

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(February 1, 2008)

Abstract

We consider the $D = 4$, $N = 8$ supergravity on $AdS_2 \times S^2$ space. We obtain the truncated Lagrangian for the bosonic chiral primary fields, and compute the tree level three-point correlation functions.

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I. INTRODUCTION

Among all known examples of the AdS/CFT correspondence [1], [2], [3], [4], the least understood is the AdS_2/CFT_1 case. The $D = 1$ conformal field theory (CFT), or conformal quantum mechanics (CQM), has not been formulated and therefore no quantitative comparison between the two sides of the duality has been made. However, there have been conjectures [5], [6], [7] that the dual CFT is given by the n-particle, $\mathcal{N} = 4$ superconformal Calogero model, which has yet to be constructed for arbitrary n [8]. By going to the second-quantized formulation, this becomes a $1 + 1$ dimensional field theory. (See also ref. [9], [10], [11], [12].)

As a first step toward investigating this conjecture we consider the supergravity side of the duality. The low energy limit of type II(A or B) string theory on T^6 is the $\mathcal{N} = 8$ supergravity theory of Cremmer and Julia [13]. Off-shell this theory has an $SO(8)$ symmetry while on-shell the symmetry is enhanced to an $E_{7(7)}$ duality. We consider the fluctuations of the fields around a classical configuration called Bertotti-Robinson solution, which gives the $AdS_2 \times S^2$ spacetime. This solution is the near-horizon limit of four D-branes intersecting over a string. The equations for the fluctuations of the fields were considered up to linear order in ref. [14], [15] to obtain the physical spectrum of the theory. [1]

In this paper, we compute the cubic interactions among the bosonic chiral primary fields. Such computations were done for $D = 10$ IIB supergravity on $AdS_5 \times S^5$ and $D = 6$ supergravity on $AdS_3 \times S^3$ in ref. [18] and [19], respectively. The computation involves the standard elements of previous works, namely nonlinear field redefinitions, which then lead to three point functions of chiral operators. The results have some similarities in form with the examples in the higher dimensional AdS spaces. It would be interesting to see whether one can make computations in a supersymmteric Calogero model, to be compared with these results.

II. SUPERGRAVITY IN $D = 4$

In this section, we review following [14], [15] the equations of motion of $D = 4$, $\mathcal{N} = 8$ supergravity. We mainly follow the four dimensional formalism and notation of ref. [14]. We are interested in the equation of motion for the metric $g_{\hat{\mu}\hat{\nu}}$, vectors $B_{\hat{\mu}}^{AB}$, and scalars W_{ABCD}, where the capital roman letters are $SU(8)$ indices. The scalars are packaged in a $E_{7(7)}$ matrix $\mathcal{V}$ parametrizing the coset $\frac{E_{7(7)}}{SU(8)}$ [13],

$$\partial_{\hat{\mu}}\mathcal{V}\mathcal{V}^{-1} = \begin{pmatrix} Q_{\hat{\mu}[A}{}^{[C}\delta_{B]}{}^{D]} & P_{\hat{\mu}ABCD} \\ \bar{P}_{\hat{\mu}}{}^{ABCD} & \bar{Q}_{\hat{\mu}}{}^{[A}{}_{[C}\delta^{B]}{}_{D]} \end{pmatrix}. \tag{2.1}$$

P's parametrize the coset manifold and can be expressed in terms of the scalar fields W, and Q's are the $SU(8)$ gauge fields, but for our purposes it is enough to note that the $SU(8)$ gauge symmetry can be fixed to the so called symmetric gauge where $\mathcal{V} = \exp(X)$,

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[1] See also ref. [16], [17] for the spectrum of the minimal $D = 4, \mathcal{N} = 2$ supergravity on $AdS_2 \times S^2$ and the $D = 10$, IIA supergravity on 'quasi' $AdS_2 \times S^8$, respectively.

$$X = \begin{pmatrix} 0 & W_{ABCD} \\ \bar{W}^{ABCD} & 0 \end{pmatrix}, \tag{2.2}$$

and W_{ABCD} is complex, completely antisymmetric in A, B, C, D, and satisfies the constraint

$$\bar{W}^{ABCD} = \frac{1}{24}\epsilon^{ABCDEFGH}W_{EFGH}. \tag{2.3}$$

We will use the following notation for indices: $\hat{\mu}, \hat{\nu} = 0, 1, 2, 3$ are $D = 4$ coordinates, $\lambda, \mu, \nu \cdots = 0, 1$ are AdS_2 coordinates and $\alpha, \beta, \gamma \cdots = 2, 3$ are S^2 coordinates.

The bosonic part of the $D = 4$ supergravity action is:

$$\mathcal{L} = \left(\frac{1}{4}eR(\omega, e) + \frac{1}{8}eF^{MN}_{\hat{\mu}\hat{\nu}}(B)\tilde{H}^{\hat{\mu}\hat{\nu}}_{MN}(B, \mathcal{V}) - \frac{1}{24}eP_{\hat{\mu}ABCD}\bar{P}^{\hat{\mu}ABCD}\right) \tag{2.4}$$

where

$$F^{AB}_{\hat{\mu}\hat{\nu}} = 2\partial_{[\hat{\mu}}B^{AB}_{\hat{\nu}]} \tag{2.5}$$

$\tilde{H}^{\hat{\mu}\hat{\nu}}_{MN}$ are defined in the Appendix 1. For our purpose, we only need the leading expansion of the $\tilde{H}$ and P in W^{ABCD}:

$$\begin{aligned} G^{MN}_{\hat{\mu}\hat{\nu}}\tilde{H}^{(B)}_{\hat{\mu}\hat{\nu}MN} &= -G^{MN}_{\hat{\mu}\hat{\nu}}(1 + W + \bar{W} + W^2 + \bar{W}^2 \\ &\quad -\frac{1}{3}W\bar{W}W - \frac{1}{3}\bar{W}W\bar{W} + W^3 + \bar{W}^3)_{MNPQ}G^{\hat{\mu}\hat{\nu}PQ} \\ &\quad +iG^{MN}_{\hat{\mu}\hat{\nu}}(W - \bar{W} + W^2 - \bar{W}^2 + O(W^3))_{MNPQ}\tilde{G}^{\hat{\mu}\hat{\nu}PQ} + O(W^4), \\ P_{\hat{\mu}ABCD} &= \nabla_{\hat{\mu}}W_{ABCD} + O(W^3). \end{aligned} \tag{2.6}$$

III. EQUATIONS OF MOTION FOR THE BOSONIC CHIRAL PRIMARY FIELDS

We consider the classical configuration called Bertotti-Robinson solution [20],

$$\begin{aligned} ds^2 &= \frac{1}{z^2}(-dx_0^2 + dz^2) + d\Omega_2^2, \\ R_{\mu\lambda\nu\sigma} &= -(g_{\mu\nu}g_{\lambda\sigma} - g_{\mu\sigma}g_{\lambda\nu}) \\ R_{\alpha\gamma\beta\delta} &= (g_{\alpha\beta}g_{\gamma\delta} - g_{\alpha\delta}g_{\gamma\beta}) \\ \bar{F}^{12}_{\alpha\beta} &= \epsilon_{\alpha\beta}, \\ \bar{F}^{AB}_{\alpha\beta} &= 0 (A \neq 1, 2), \\ W_{ABCD} &= 0 \end{aligned} \tag{3.1}$$

which breaks the $SU(8)$ internal symmetry into $SU(6) \times SU(2)$. The bulk fields of interest are the fluctuations about this background,

$$\begin{aligned} g_{\hat{\mu}\hat{\nu}} &= \bar{g}_{\hat{\mu}\hat{\nu}} + h_{\hat{\mu}\hat{\nu}}, \\ F^{AB}_{\hat{\mu}\hat{\nu}} &= \bar{F}^{AB_{\hat{\mu}\hat{\nu}}} + 2\nabla_{[\hat{\mu}}b^{AB}_{\hat{\nu}]} \end{aligned} \tag{3.2}$$

and W_{ABCD}. One can consider the classical equations for these fluctuations to linear order and organize the spectrum into the multiplets of $SU(6) \times SU(2)$ [14], [15]. In this paper we expand the equations of motion up to the second order in these fluctuations to obtain the interaction terms.

To find the chiral primary fields on AdS_2 we expand the fields in spherical harmonics on S^2. The expansions are quite simple in this case as all harmonic functions on the 2-sphere can be expressed in terms of just the scalar spherical harmonics Y_{lm}. l is the quantum number labeling the Casimir of the representation,

$$\nabla_\alpha \nabla^\alpha Y_{lm} = -l(l+1) Y_{lm}. \tag{3.3}$$

The expansions of the bosonic fluctuations are then given by (denoting the l, m indices collectively by I)

$$\begin{aligned}
h_{\mu\nu} &= \sum H^I_{\mu\nu} Y_I && (3.4a)\\
h_{\mu\alpha} &= \sum (B^I_{1\mu} \nabla_\alpha Y_I + B^I_{2\mu} e_{\alpha\beta} \nabla^\beta Y_I) && (3.4b)\\
h_{\alpha\beta} &= \sum (\phi^I_1 \nabla_\alpha \nabla_\beta Y_I + \phi^I_2 e_{(\alpha}{}^\gamma \nabla_{\beta)} \nabla_\gamma Y_I + \phi^I_3 g_{\alpha\beta} Y_I) && (3.4c)\\
b^{AB}_\mu &= \sum b^{(I)AB}_\mu Y_I && (3.4d)\\
b^{AB}_\alpha &= \sum (b^{(I)AB}_1 \nabla_\alpha Y_I + b^{(I)AB}_2 e_{\alpha\beta} \nabla^\beta Y_I) && (3.4e)\\
W_{ABCD} &= \sum W^I_{ABCD} Y_I. && (3.4f)
\end{aligned}$$

Before substituting the expansions into the equations of motion we can first simplify the expansions by fixing gauge symmetries by imposing[2]

$$\phi^I_1 = \phi^I_2 = B^I_{1\mu} = b^{(I)AB}_1 = 0. \tag{3.5}$$

At the linearized level, the bosonic fields can be decomposed into the eigenstates of the AdS_2 Laplacian [14], [15],

$$\begin{aligned}
\phi^I &= 2lT^I + 2(l-1)\tilde{T}^I \\
b^{(I)[12]} &= T^I - \tilde{T}^I \\
a^{(I)[12]} &\equiv \epsilon^{\mu\nu} \nabla_\mu b^{(I)[12]}_\nu = (l+1) S^I + l \tilde{S}^I \\
B^I &\equiv \epsilon^{\mu\nu} \nabla_\mu B^I_\nu = -2S^I + 2\tilde{S}^I \\
a^{(I)[MN]} &\equiv \epsilon^{\mu\nu} \nabla_\mu b^{(I)[MN]}_\nu = (l-1) U^{(I)[MN]} + (l+2) \tilde{U}^{(I)[MN]} \quad (M, N \neq 1, 2) \\
(W - \bar{W})^{(I)12MN} &= i \nabla^2_x (U^{(I)[MN]} - \tilde{U}^{(I)[MN]}) \\
b^{(I)[MN]} &= V^{(I)[MN]} + \tilde{V}^{(I)[MN]} \quad (M, N \neq 1, 2) \\
(W + \bar{W})^{(I)12MN} &= -lV^{(I)[MN]} + (l+1) \tilde{V}^{(I)[MN]}
\end{aligned} \tag{3.6}$$

where these fields satisify the equations of the form:

[2] Since we project out only the physical modes, actually it is enough to impose $\phi_1 = \phi_2 = 0$. From now on, we write $\phi_3 \to \phi, B_{2\mu} \to B_\mu, b_2 \to b$.

$$(\nabla_x^2 - l(l-1))A^I + Q_A^I = 0 \tag{3.7}$$

$$(\nabla_x^2 - (l+1)(l+2))\tilde{A}^I + \tilde{Q}_{\tilde{A}}^I = 0 \tag{3.8}$$

Here, A and $\tilde{A}$ stand for T, S, U, V and $\tilde{T}, \tilde{S}, \tilde{U}, \tilde{V}$ respectively, and Q_A and $Q_{\tilde{A}}$ are the second order corrections. Also, we note that the fields $H_{\mu\nu}$ are not independent degrees of freedom and they are completely determined by the equations [14], [15], [16]

$$\Big((\nabla_x^2 + 2 - l(l+1))H_{\mu\nu}^I - 2\nabla_{(\mu}\nabla^\lambda H_{\nu)\lambda}^I + (\nabla_\mu\nabla_\nu - g_{\mu\nu}(\nabla_x^2 + 1 - l(l+1)))H^I$$
$$+g_{\mu\nu}\nabla^\lambda\nabla^\rho H_{\lambda\rho}^I + 2(\nabla_\mu\nabla_\nu - g_{\mu\nu}(\nabla_x^2 - 1 - \frac{1}{2}l(l+1)))\phi^I\Big) = 4g_{\mu\nu}l(l+1)b^I + (\text{higher order}) \tag{3.9a}$$

$$(\nabla^\nu H_{\mu\nu}^I - \nabla_\mu H^I - \nabla_\mu\phi^I) = -4\nabla_\mu b^I + (\text{higher order}) \tag{3.9b}$$

$$\Big((\nabla_x^2 + 4)\phi^I + (\nabla_x^2 - 1 - l(l+1))H^I - \nabla^\mu\nabla^\nu H_{\mu\nu}^I\Big) = 4l(l+1)b^I + (\text{higher order}) \tag{3.9c}$$

$$H^I = (\text{quadratic order}). \tag{3.9d}$$

By making an ansatz, one can easily find the solution to the equations above:

$$H_{\mu\nu}^I = \frac{1}{l+1}\left(-2l(l-1)g_{\mu\nu}T^I + 4\nabla_\mu\nabla_\nu T^I\right) + (\text{higher order corrections}) \tag{3.10}$$

Since we are interested only in the three-point function of chiral primary fields, only Q_T, Q_S, Q_U, Q_V are of interest. We also put any non-chiral primary fields appearing in Q's to be zero, and substitute (3.10) for $H_{\mu\nu}$, neglecting the higher order corrections. The detailed form of the Q's are rather complicated and not interesting at this stage. They are of the generic form:

$$Q_A = \alpha\nabla^\mu\nabla_\nu B\nabla_\mu\nabla_\nu C + \beta\nabla^\mu B\nabla_\mu C + \gamma BC + \cdots, \tag{3.11}$$

where A, B, C are chiral primary fields. We see that there are terms involving derivatives of the fields in Q's. These terms can be removed by nonlinear redefinitions of the fields, which does not change the equations (3.8) at the linear level. It is easy to see that this can be done by redefining [18], [19]

$$A \to A - \frac{\alpha}{2c}(\nabla B)\cdot(\nabla C) - \frac{1}{2c}(\alpha + (1+\Gamma)\beta)B\cdot C, \tag{3.12}$$

where $\Gamma \equiv \frac{1}{2}(l(l-1) - l_1(l_1-1) - l_2(l_2-1))$ with l, l_1, l_2 being the total angular momentum quantum numbers of A, B, C, respectively. After these redefinition, the linear term

$$(\nabla_x^2 - l(l-1))A \tag{3.13}$$

generates additional term which removes the derivative terms, and Q_A becomes:

$$Q_A = (\gamma + \Gamma(\beta + (1+\Gamma)\alpha))BC\cdots \tag{3.14}$$

We then get

$$
\begin{aligned}
Q_T =& \frac{(l(l^2-1)+l_1(l_1^2-1)+l_2(l_2^2-1))}{2l(2l+1)(l-1)(l_1+1)(l_2+1)}\alpha\alpha_1\alpha_2(\Sigma^2-1)\tilde{C}(I;I_1,I_2)T^2 \\
&-\frac{(l(l^2-1)-l_1(l_1^2-1)-l_2(l_2^2-1))\tilde{C}(I;I_1,I_2)}{2l(2l+1)(l-1)}\alpha\alpha_1\alpha_2(\Sigma^2-1)S^2 \\
&+\frac{(l_1+l_2)(l_1-1)(l_2-1)}{4l(2l+1)(l-1)}\alpha\alpha_1\alpha_2(\Sigma^2-1)\tilde{C}(I;I_1,I_2)\epsilon^{12ABCDEF}U_{CD}U_{EF} \\
&+\frac{(l_1+l_2)}{4l(2l+1)(l-1)}\alpha\alpha_1\alpha_2(\Sigma^2-1)\tilde{C}(I;I_1,I_2)\epsilon^{12ABCDEF}V_{CD}V_{EF}
\end{aligned} \tag{3.15}
$$

$$
\begin{aligned}
Q_S =& \frac{(l(l^2-1)+l_1(l_1^2-1)-l_2(l_2^2-1))}{2l(l^2-1)(2l+1)(l_2+1)}\alpha\alpha_1\alpha_2(\Sigma^2-1)\tilde{C}(I;I_1,I_2)S^{(1)}T^{(2)} \\
&-\frac{(l_1-l_2)(l_2-1)}{4l(l^2-1)(2l+1)}\alpha\alpha_1\alpha_2(\Sigma^2-1)\tilde{C}(I;I_1,I_2)V^{(1)}U^{(2)}
\end{aligned} \tag{3.16}
$$

$$
\begin{aligned}
Q_U^{AB} =& \frac{(l-l_2)}{2l(l-1)(2l+1)}\alpha\alpha_1\alpha_2(\Sigma^2-1)\tilde{C}(I;I_1,I_2)S^{(1)}V^{(2)AB} \\
&-\frac{l_1-1}{8l(l-1)(2l+1)}\alpha\alpha_1\alpha_2(\Sigma^2-1)\tilde{C}(I;I_1,I_2)\epsilon^{12ABCDEF}U_{CD}^{(1)}V_{EF}^{(2)} \\
&+\frac{(l+l_1)(l_1-1)}{2l(l-1)(2l+1)(1+l_2)}\alpha\alpha_1\alpha_2(\Sigma^2-1)\tilde{C}(I;I_1,I_2)U^{(1)AB}T^{(2)}
\end{aligned} \tag{3.17}
$$

$$
\begin{aligned}
Q_V^{AB} =& \frac{(l_2-l)(l_2-1)}{2l(2l+1)}\alpha\alpha_1\alpha_2(\Sigma^2-1)\tilde{C}(I;I_1,I_2)S^{(1)}U^{(2)AB} \\
&-\frac{(l_1-1)(l_2-1)}{16l(2l+1)}\alpha\alpha_1\alpha_2(\Sigma^2-1)\tilde{C}(I;I_1,I_2)\epsilon^{12ABCDEF}U_{CD}U_{EF} \\
&+\frac{1}{16l(2l+1)}\alpha\alpha_1\alpha_2(\Sigma^2-1)\tilde{C}(I;I_1,I_2)\epsilon^{12ABCDEF}V_{CD}V_{EF} \\
&+\frac{(l+l_1)}{2l(2l+1)(1+l_2)}\alpha\alpha_1\alpha_2(\Sigma^2-1)\tilde{C}(I;I_1,I_2)V^{(1)AB}T^{(2)}
\end{aligned} \tag{3.18}
$$

at the quadratic level, where $\alpha \equiv l_1+l_2-l$, $\alpha_1 \equiv l+l_2-l_1$, $\alpha_2 \equiv l+l_1-l_2$, $\Sigma \equiv l+l_1+l_2$, and

$$
\tilde{C}(I;I_1,I_2) \equiv \int Y_I^* Y_{I_1} Y_{I_2}. \tag{3.19}
$$

We also used the abbreviation $A^{(i)} \equiv A^{I_i}$ and $A^2 \equiv A^{I_1}A^{I_2}$ for any field A.

IV. THE EFFECTIVE ACTION AND THE THREE-POINT FUNCTION FUNCTION

The equations in the previous section can be derived from the truncated Lagrangian

$$
\mathcal{L} = \left(\frac{1}{4}eR(\omega,e) + \frac{1}{8}eF^{MN}_{\hat{\mu}\hat{\nu}}(B)\tilde{H}^{\hat{\mu}\hat{\nu}}_{MN}(B,\mathcal{V}) - \frac{1}{24}eP_{\hat{\mu}ABCD}\bar{P}^{\hat{\mu}ABCD}\right)
$$

$$
\begin{aligned}
&= \frac{(2l+1)l(l^2-1)}{4} T(\nabla_x^2 - l(l-1))T + \frac{(2l+1)l(l^2-1)}{2} S(\nabla_x^2 - l(l-1))S \\
&+ \frac{l(2l+1)}{4} V^{AB}(\nabla_x^2 - l(l-1))V_{AB} + \frac{l(l-1)^2(2l+1)}{4} U^{AB}(\nabla_x^2 - l(l-1))U_{AB} \\
&+ \frac{(l_1(l_1^2-1) + l_2(l_2^2-1) + l_3(l_3^2-1))}{3(l_1+1)(l_2+1)(l_3+1)} \alpha_1\alpha_2\alpha_3(\Sigma^2-1)C(I_1,I_2,I_3)T^3 \\
&+ \frac{(l_1(l_1^2-1) + l_2(l_2^2-1) - l_3(l_3^2-1))}{(l_3+1)} \alpha_1\alpha_2\alpha_3(\Sigma^2-1)C(I_1,I_2,I_3)S^{(1)}S^{(2)}T^{(3)} \\
&-(l_1-l_2)(l_2-1)\alpha_1\alpha_2\alpha_3(\Sigma^2-1)C(I_1,I_2,I_3)V^{(1)AB}U^{(2)}_{AB}S^{(3)} \\
&+ \frac{1}{24}\alpha_1\alpha_2\alpha_3(\Sigma^2-1)C(I_1,I_2,I_3)\epsilon^{12ABCDEF}V^{(1)}_{AB}V^{(2)}_{CD}V^{(3)}_{EF} \\
&+ \frac{l_1+l_2}{2(l_3+1)}\alpha_1\alpha_2\alpha_3(\Sigma^2-1)C(I_1,I_2,I_3)V^{(1)AB}V^{(2)}_{AB}T^{(3)} \\
&- \frac{(l_1-1)(l_2-1)}{8}\alpha_1\alpha_2\alpha_3(\Sigma^2-1)C(I_1,I_2,I_3)\epsilon^{12ABCDEF}U^{(1)}_{AB}U^{(2)}_{CD}V^{(3)}_{EF} \\
&+ \frac{(l_1-1)(l_2-1)(l_1+l_2)}{2(l_3+1)}\alpha_1\alpha_2\alpha_3(\Sigma^2-1)C(I_1,I_2,I_3)U^{(1)AB}U^{(2)}_{AB}T^{(3)}. \qquad (4.1)
\end{aligned}
$$

with

$$
C(I_1,I_2,I_3) \equiv \int Y^{I_1}Y^{I_2}Y^{I_3}. \qquad (4.2)
$$

Here the normalizations were fixed by directly substituting the expression (3.6) into the lagrangian (2.4) and evaluating the leading terms for some fields.

To compute from (4.1) the 2- and 3-point functions of chiral primary operators of the boundary theory, we apply the formulae derived, for instance, in ref. [21]. From Eq.(17) and the correction factor in Eq.(95) of ref. [21], we read off the tree-level two-point functions to be[3]

$$
\begin{aligned}
\langle T^{I_1}(x)T^{I_2}(y)\rangle &= \frac{(2l+1)(l^2-1)}{2}\frac{1}{\pi^{1/2}}\frac{\Gamma(l+1)}{\Gamma(l-1/2)}(2l-1)\frac{\delta^{I_1I_2}}{|x-y|^{2l}}. \\
\langle S^{I_1}(x)S^{I_2}(y)\rangle &= (2l+1)(l^2-1)\frac{1}{\pi^{1/2}}\frac{\Gamma(l+1)}{\Gamma(l-1/2)}(2l-1)\frac{\delta^{I_1I_2}}{|x-y|^{2l}}.
\end{aligned}
$$

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[3] Although the following results are the correlation functions of the chiral primary operators of the boundary theory which couple to the bulk fields S, T, U, V we will still denote them by S, T, U, V for the simplicity of the notations.

$$\langle U^{(I_1)AB}(x)U^{(I_2)CD}(y)\rangle = (l-1)^2(2l+1)\frac{1}{\pi^{1/2}}\frac{\Gamma(l+1)}{\Gamma(l-1/2)}(2l-1)\frac{(\delta_{AC}\delta_{BD}-\delta_{AD}\delta_{BC})\delta^{I_1I_2}}{|x-y|^{2l}}.$$
$$\langle V^{(I_1)AB}(x)V^{(I_2)CD}(y)\rangle = (2l+1)\frac{1}{\pi^{1/2}}\frac{\Gamma(l+1)}{\Gamma(l-1/2)}(2l-1)\frac{(\delta_{AC}\delta_{BD}-\delta_{AD}\delta_{BC})\delta^{I_1I_2}}{|x-y|^{2l}}. \qquad (4.3)$$

From Eq.(25) of the same paper we derive that

$$\langle T^{(1)}(x)T^{(2)}(y)T^{(3)}(z)\rangle = -\frac{2^5}{\pi}\frac{\prod_i\Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})(l_1(l_1^2-1)+l_2(l_2^2-1)+l_3(l_3^2-1))}{\prod_i^3\left(\Gamma(l_i-1/2)(l_i+1)\right)|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}C(I_1,I_2,I_3)$$
$$\langle S^{(1)}S^{(2)}T^{(3)}\rangle = -\frac{2^5}{\pi}\frac{\prod_i\Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})(l_1(l_1^2-1)+l_2(l_2^2-1)+l_3(l_3^2-1))}{\prod_i^3\left(\Gamma(l_i-1/2)\right)(l_3+1)|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}C(I_1,I_2,I_3)$$
$$\langle V^{(1)AB}U^{(2)CD}S^{(3)}\rangle = \frac{2^5}{\pi}\frac{\prod_i\Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})(l_1-l_2)(l_2-1)}{\prod_i^3\left(\Gamma(l_i-1/2)\right)|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}$$
$$\times(\delta^{AC}\delta^{BD}-\delta^{AD}\delta^{BC})C(I_1,I_2,I_3)$$

$$\langle V^{(1)}_{AB}V^{(2)}_{CD}V^{(3)}_{EF}\rangle = -\frac{2^5}{\pi}\frac{\prod_i\Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})}{\prod_i^3\left(\Gamma(l_i-1/2)\right)|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}C(I_1,I_2,I_3)\epsilon_{12ABCDEF}$$
$$\langle V^{(1)}_{AB}V^{(2)}_{CD}T^{(3)}\rangle = -\frac{2^6}{\pi}\frac{(l_1+l_2)\prod_i\Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})(\delta_{AC}\delta_{BD}-\delta_{AD}\delta_{BC})}{(l_3+1)\prod_i^3\left(\Gamma(l_i-1/2)\right)|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}C(I_1,I_2,I_3)$$
$$\langle U^{(1)}_{AB}U^{(2)}_{CD}V^{(3)}_{EF}\rangle = \frac{2^5}{\pi}\frac{(l_1-1)(l_2-1)\prod_i\Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})}{\left(\prod_i^3\Gamma(l_i-1/2)\right)|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}C(I_1,I_2,I_3)\epsilon_{12ABCDEF}$$
$$\langle U^{(1)AB}U^{(2)CD}T^{(3)}\rangle = -\frac{2^6}{\pi}\frac{(l_1-1)(l_2-1)(l_1+l_2)\prod_i\Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})}{(l_3+1)\left(\prod_i^3\Gamma(l_i-1/2)\right)|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}$$
$$\times(\delta^{AC}\delta^{BD}-\delta^{AD}\delta^{BC})C(I_1,I_2,I_3). \qquad (4.4)$$

The normalizations of the fields can be fixed by demanding that the two point functions are

$$\langle A^{(I_1)}(x)A^{(I_2)}(y)\rangle = \frac{\delta^{I_1I_2}}{|x-y|^{2l}}. \qquad (4.5)$$

for any two chiral primary fields A^I. After rescaling the fields in order to satisfy this condition, the normalized three-point function are given by:

$$\langle T^{(1)}(x)T^{(2)}(y)T^{(3)}(z)\rangle = -\frac{2^{7/2}}{\pi^{1/4}}\frac{\prod_i\Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})}{\sqrt{\prod_i^3\left(\Gamma(l_i+3/2)\Gamma(l_i+2)(l_i+1)(l_i^2-1)\right)}}$$
$$\times\frac{(l_1(l_1^2-1)+l_2(l_2^2-1)+l_3(l_3^2-1))}{|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}C(I_1,I_2,I_3)$$
$$\langle S^{(1)}S^{(2)}T^{(3)}\rangle = -\frac{2^{5/2}}{\pi^{1/4}}\frac{\prod_i\Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})}{\sqrt{\prod_i^3\left(\Gamma(l_i+3/2)\Gamma(l_i+1)(l_i^2-1)\right)(l_3+1)}}$$

$$\times \frac{(l_1(l_1^2-1)+l_2(l_2^2-1)+l_3(l_3^2-1))}{|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}} C(I_1,I_2,I_3)$$

$$\langle V^{(1)AB}U^{(2)CD}S^{(3)}\rangle = \frac{2^2}{\pi^{1/4}} \frac{\prod_i \Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})(l_1-l_2)C(I_1,I_2,I_3)(\delta^{AC}\delta^{BD}-\delta^{AD}\delta^{BC})}{\sqrt{\prod_i^3 (\Gamma(l_i+3/2)\Gamma(l_i+1))(l_3^2-1)}|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}$$

$$\langle V^{(1)}_{AB}V^{(2)}_{CD}V^{(3)}_{EF}\rangle = -\frac{2^2}{\pi^{1/4}} \frac{\prod_i \Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})C(I_1,I_2,I_3)\epsilon_{12ABCDEF}}{\sqrt{\prod_i^3 (\Gamma(l_i+3/2)\Gamma(l_i+1))}|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}$$

$$\langle V^{(1)}_{AB}V^{(2)}_{CD}T^{(3)}\rangle = -\frac{2^{7/2}}{\pi^{1/4}} \frac{(l_1+l_2)\prod_i \Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})}{(l_3+1)\sqrt{\prod_i^3 (\Gamma(l_i+3/2)\Gamma(l_i+1))(l_3^2-1)}}$$

$$\times \frac{(\delta_{AC}\delta_{BD}-\delta_{AD}\delta_{BC})C(I_1,I_2,I_3)}{|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}$$

$$\langle U^{(1)}_{AB}U^{(2)}_{CD}V^{(3)}_{EF}\rangle = \frac{2^2}{\pi^{1/4}} \frac{\prod_i \Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})C(I_1,I_2,I_3)\epsilon_{12ABCDEF}}{\sqrt{\left(\prod_i^3 \Gamma(l_i+3/2)\Gamma(l_i+1)\right)}|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}$$

$$\langle U^{(1)AB}U^{(2)CD}T^{(3)}\rangle = -\frac{2^{7/2}}{\pi^{1/4}} \frac{(l_1+l_2)\prod_i \Gamma(\frac{\alpha_i}{2}+1)\Gamma(\frac{1}{2}\Sigma+\frac{3}{2})}{(l_3+1)\sqrt{\left(\prod_i^3 \Gamma(l_i+3/2)\Gamma(l_i+1)\right)(l_3^2-1)}}$$

$$\times \frac{(\delta^{AC}\delta^{BD}-\delta^{AD}\delta^{BC})C(I_1,I_2,I_3)}{|x-y|^{\alpha_3}|y-z|^{\alpha_1}|z-x|^{\alpha_2}}. \tag{4.6}$$

V. CONCLUSIONS

In this paper, we gave the calculation of three point interactions of chiral primaries in 4D supergravity. In order to remove the derivatives from the interactions, we used nontrivial redefinitions of the fields which solved the linear equation of motion for chiral primaries. It is after removing the derivative terms that we deal with a standard field theory in two dimensions. These redefinitions were also needed for the cases of higher dimensional AdS spaces [18], [19]. Deeper reasons behind these redefinitions are still not clear. [4]

In section 4, we derived the three point interactions. The factors appear which are similar to the results in higher dimensional AdS spacetimes. In order to make a useful statement on AdS_2/CFT_1 correspondence, similar computation has to be made in the boundary conformal quantum mechanics dual to this theory. There has been a conjecture that this dual quantum mechanics is given by a supersymmetric extension of the Calogero model [5], [6], [7]. It would be interesting to see whether this is true. However, since a many-body quantum mechanics becomes a $1+1$ dimensional field theory after the second quantization, one might directly obtain this field theory starting from the Lagrangian (4.1) by appropriate truncation. This

[4] See also ref. [22] for related discussions for the case of the $D = 11$ supergravity on $AdS_7 \times S^4$.

interesting issue, along with the evluation of the interaction terms for the fermions, are left for future studies.

Acknowledgements: I give special thanks to Antal Jevicki for many helpful discussions and suggestions. I also thank Tamiaki Yoneya and Sangmin Lee for useful discussions. This work was supported by JSPS through Institute of Physics, University of Tokyo.

Appendix 1 : Vector and scalar part of the lagrangian As mentioned in the text, the $SU(8)$ gauge symmetry can be fixed to the so called symmetric gauge where $\mathcal{V} = \exp(X)$ and

$$X = \begin{pmatrix} 0 & W_{ABCD} \\ \bar{W}^{ABCD} & 0 \end{pmatrix} \tag{1}$$

Expanding in W, we get the expressions

$$\mathcal{V} = \begin{pmatrix} \delta_{AB}{}^{CD} + \frac{1}{2}W_{ABEF}\bar{W}^{EFCD} + O(W^4) & W_{ABCD} + O(W^3) \\ \bar{W}^{ABCD} + O(W^3) & \delta^{AB}{}_{CD} + \frac{1}{2}\bar{W}^{ABEF}W_{EFCD} + O(W^4) \end{pmatrix}, \tag{2}$$

and

$$\mathcal{V}^{-1} = \begin{pmatrix} \delta_{AB}{}^{CD} + \frac{1}{2}W_{ABEF}\bar{W}^{EFCD} + O(W^4) & -W_{ABCD} + O(W^3) \\ -\bar{W}^{ABCD} + O(W^3) & \delta^{AB}{}_{CD} + \frac{1}{2}\bar{W}^{ABEF}W_{EFCD} + O(W^4) \end{pmatrix}. \tag{3}$$

From these expressions one easily gets

$$\partial_{\hat{\mu}}\mathcal{V}\mathcal{V}^{-1} = \begin{pmatrix} \frac{1}{2}\nabla_\mu(W\bar{W}) - (\nabla_\mu W)\bar{W} + O(W^4) & \nabla_\mu W + O(W^3) \\ \nabla_\mu \bar{W} + O(W^3) & \frac{1}{2}\nabla_\mu(\bar{W}W) - (\nabla_\mu \bar{W})W + O(W^4) \end{pmatrix} \tag{4}$$

Comparing with Eq.(2.1), we see that

$$P_{\hat{\mu}ABCD} = \nabla_{\hat{\mu}}W_{ABCD} + O(W^3), \quad \bar{P}^{\hat{\mu}ABCD} = \nabla_{\hat{\mu}}W^{ABCD} + O(W^3). \tag{5}$$

$\tilde{H}(B, \mathcal{V})$ is defined by the equation

$$\begin{pmatrix} G_{\hat{\mu}\hat{\nu}} + iH_{\hat{\mu}\hat{\nu}} \\ G_{\hat{\mu}\hat{\nu}} - iH_{\hat{\mu}\hat{\nu}} \end{pmatrix} = (\mathcal{V}^\dagger\mathcal{V})^{-1} \begin{pmatrix} i\tilde{G}_{\hat{\mu}\hat{\nu}} - \tilde{H}_{\hat{\mu}\hat{\nu}} \\ -i\tilde{G}_{\hat{\mu}\hat{\nu}} - \tilde{H}_{\hat{\mu}\hat{\nu}} \end{pmatrix}, \tag{6}$$

where it is to be understood that the matrices are multiplied by contracting the $SU(8)$ indices, which I did not write explicitly. [5] $\tilde{G}, \tilde{H}$ denotes the dual fields,

$$\tilde{G}^{\hat{\mu}\hat{\nu}} = \frac{1}{2}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}G_{\hat{\rho}\hat{\sigma}}, \quad \tilde{H}^{\hat{\mu}\hat{\nu}} = \frac{1}{2}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}H_{\hat{\rho}\hat{\sigma}} \tag{7}$$

Expanding in W, we get

---

[5] In this paper, I use only $SU(8)$ indices, in contrast to ref. [14] where E_7 indices were also used.

$$\begin{pmatrix} G_{\hat{\mu}\hat{\nu}} + iH_{\hat{\mu}\hat{\nu}} \\ G_{\hat{\mu}\hat{\nu}} - iH_{\hat{\mu}\hat{\nu}} \end{pmatrix} = \begin{pmatrix} 1 + 2W\bar{W} + O(W^4) & -2W - \frac{4}{3}W\bar{W}W + O(W^5) \\ -2\bar{W} - \frac{4}{3}\bar{W}W\bar{W} + O(W^5) & 1 + 2\bar{W}W + O(W^4) \end{pmatrix} \begin{pmatrix} i\tilde{G}_{\hat{\mu}\hat{\nu}} - \tilde{H}_{\hat{\mu}\hat{\nu}} \\ -i\tilde{G}_{\hat{\mu}\hat{\nu}} - \tilde{H}_{\hat{\mu}\hat{\nu}} \end{pmatrix}. \tag{8}$$

By adding the first and second row, we get

$$\begin{aligned} G = &-(1 - W - \bar{W} + W\bar{W} + \bar{W}W - \frac{2}{3}\bar{W}W\bar{W} - \frac{2}{3}W\bar{W}W + O(W^5))\tilde{H} \\ &+i(-\bar{W} + W + W\bar{W} - \bar{W}W - \frac{2}{3}\bar{W}W\bar{W} + \frac{2}{3}W\bar{W}W)\tilde{G}. \end{aligned} \tag{9}$$

Solving in terms of H, we finally get

$$\begin{aligned} G^{AB}_{\hat{\mu}\hat{\nu}}\tilde{H}^{(B)}_{\hat{\mu}\hat{\nu}AB} = &-G^{AB}_{\hat{\mu}\hat{\nu}}(1 + W + \bar{W} + W^2 + \bar{W}^2 \\ &-\frac{1}{3}W\bar{W}W - \frac{1}{3}\bar{W}W\bar{W} + W^3 + \bar{W}^3)_{ABCD}G^{\hat{\mu}\hat{\nu}CD} \\ &+iG^{AB}_{\hat{\mu}\hat{\nu}}(W - \bar{W} + W^2 - \bar{W}^2 + O(W^3))_{ABCD}\tilde{G}^{\hat{\mu}\hat{\nu}CD} + O(W^4), \\ &. \end{aligned} \tag{10}$$

Appendix 2 : Spherical Harmonics

We list some formulas about spherical harmonics needed for the calculations in the text. The spherical hamonics on S^2 are very simple in that there are only scalar spherical harmonics, which we denote by $Y_I = Y_{lm}$. They are normalized so that

$$\int Y_{l_1 m_1} Y_{l_2 m_2} = \delta_{l_1 l_2}\delta_{m_1 m_2}. \tag{11}$$

The explicit form is given by

$$Y_{lm} \equiv \frac{1}{2}\sqrt{\frac{(2l+1)(l-m)}{\pi(l+m)}}e^{im\phi}P^m_l(cos\theta) \tag{12}$$

where $P^m_l(x)$ is the associated Legendre polynomial

$$P^m_l(x) \equiv \frac{(-1)^m}{2^l l!}(1-x^2)^{m/2}\frac{d^{m+l}}{dx^{m+l}}(x^2-1)^l, \tag{13}$$

although this explicit form was not really used in the text. More important is the formula for the integral of three spherical harmonics with derivatives, expressed in terms of $C(I_1, I_2, I_3)$. They are as follows: [6]

$$\begin{aligned} A(1;2,3) &\equiv \int Y_1 \nabla Y_2 \nabla Y_3 \\ &= \frac{1}{2}(l_2(l_2+1) + l_3(l_3+1) - l_1(l_1+1))C(1,2,3) \end{aligned} \tag{14a}$$

---

[6] again, we use the abbreviation i for $I_i \equiv (l_i, m_i)$

$$
\begin{aligned}
B(1;2,3) \equiv & \int Y_1 \nabla_\alpha \nabla_\beta Y_2 \nabla^\alpha \nabla^\beta Y_3 \\
= & \frac{1}{4}(l_1(l_1+1) - l_2(l_2+1) - l_3(l_3+1)) \\
& \times(2 + l_1(l_1+1) - l_2(l_2+1) - l_3(l_3+1))C(1,2,3) \qquad (14b) \\
D(1;2,3) \equiv & \int \nabla_\alpha \nabla_\beta Y_1 \nabla^\alpha Y_2 \nabla^\beta Y_3 \\
= & \frac{1}{4}(l_3(l_3+1) + l_1(l_1+1) - l_2(l_2+1)) \\
& \times(l_3(l_3+1) + l_2(l_2+1) - l_1(l_1+1))C(1,2,3) \qquad (14c) \\
G(1,2,3) \equiv & \int \nabla_\alpha \nabla_\beta Y_1 \nabla_\beta \nabla_\gamma Y_2 \nabla^\gamma \nabla^\alpha Y_3 \\
= & \frac{1}{8}(-l_1^3(l_1+1)^3 - l_2^3(l_2+1)^3 - l_3^3(l_3+1)^3 \\
& -2l_1^2(1_1+1)^2 - 2l_2^2(1_2+1)^2 - 2l_3^2(1_3+1)^2 \\
& +4l_1l_2(l_1+1)(l_2+1) + 4l_2l_3(l_2+1)(l_3+1) + 4l_3l_1(l_3+1)(l_1+1) \\
& +l_1^2l_2(l_1+1)^2(l_2+1) + l_1l_2^2(l_1+1)(l_2+1)^2 + l_2^2l_3(l_2+1)^2(l_3+1) \\
& +l_2l_3^2(l_2+1)(l_3+1)^2 + l_3^2l_1(l_3+1)^2(l_1+1) + l_3l_1^2(l_3+1)(l_1+1)^2 \\
& -2l_1l_2l_3(l_1+1)(l_2+1)(l_3+1)) \qquad (14d)
\end{aligned}
$$

proof)
a) By integrating by parts, one gets

$$
\begin{aligned}
A(1;2,3) &= -\int Y_1 \nabla^2 Y_2 Y_3 - \int \nabla_\alpha Y_1 \nabla^\alpha Y_2 Y_3 \\
&= l_2(l_2+1)C(1,2,3) - A(3;1,2). \qquad (15)
\end{aligned}
$$

By permuting the indices and adding the resulting equations, we get Eq.(14a).
b) By integrating by parts, we get

$$
\begin{aligned}
B(1;2,3) = & \int Y_1 \nabla_\alpha \nabla_\beta Y_2 \nabla^\alpha \nabla^\beta Y_3 \\
= & -\int \nabla_\alpha Y_1 \nabla^\alpha \nabla^\beta Y_2 \nabla_\beta Y_3 - \int Y_1 \nabla_\alpha \nabla_\beta \nabla^\alpha Y_2 \nabla^\beta Y_3 \\
= & \int \nabla_\alpha Y_1 \nabla^\alpha Y_2 \nabla_y^2 Y_3 + \nabla_\beta \nabla_\alpha Y_1 \nabla^\alpha Y_2 \nabla^\beta Y_3 \\
& -Y_1 \nabla^\gamma Y_2 \nabla^\beta Y_3 R_{\beta\gamma} - Y_1 \nabla^\beta \nabla_y^2 Y_2 \nabla_\beta Y_3 \\
= & -l_3(l_3+1)\nabla_\alpha Y_1 \nabla^\alpha Y_2 Y_3 - \nabla_\beta \nabla_\alpha \nabla^\beta Y_1 \nabla^\alpha Y_2 Y_3 - \nabla_\beta \nabla_\alpha Y_1 \nabla^\beta \nabla^\alpha Y_2 Y_3 \\
& -Y_1 \nabla^\beta Y_2 \nabla_\beta Y_3 + l_2(l_2+1) Y_1 \nabla^\beta Y_2 \nabla_\beta Y_3 \\
= & -l_3(l_3+1)A(3;1,2) - R_{\alpha\gamma} \nabla^\alpha Y_1 \nabla^\gamma Y_2 Y_3 \\
& -\nabla_\alpha \nabla_y^2 Y_1 \nabla^\alpha Y_2 Y_3 - B(3;1,2) \\
& +A(1;2,3)(l_2(l_2+1) - 1) \\
= & (l_1(l_1+1) - l_3(l_3+1) - 1)A(3;1,2) - B(3;1,2) + A(1;2,3)(l_2(l_2+1) - 1) \quad (16)
\end{aligned}
$$

which gives

$$B(1;2,3)+B(3;1,2)=(l_2(l_2+1)-1)A(1;2,3)+(l_1(l_1+1)-l_3(l_3+1)-1)A(3;1,2) \tag{17}$$

On the other hand, we have

$$\begin{aligned}
B(1;2,3) &= -\int \nabla^\alpha Y_1 \nabla_\alpha \nabla_\beta Y_2 \nabla^\beta Y_3 - \int Y_1 \nabla^\alpha \nabla_\beta \nabla_\alpha Y_2 \nabla^\beta Y_3 \\
&= \int \nabla^\alpha \nabla^\beta Y_1 \nabla_\alpha \nabla_\beta Y_2 Y_3 + \int \nabla^\alpha Y_1 \nabla^\beta \nabla_\alpha \nabla_\beta Y_2 Y_3 \\
&\quad - R_{\beta\gamma} \int Y_1 \nabla^\gamma Y_2 \nabla^\beta Y_3 - Y_1 \nabla^\beta \nabla_y^2 Y_2 \nabla_\beta Y_3 \\
&= B(3;1,2) + R_{\alpha\gamma} \nabla^\alpha Y_1 \nabla^\gamma Y_2 Y_3 \\
&\quad + \nabla_\alpha Y_1 \nabla^\alpha \nabla_y^2 Y_2 Y_3 - Y_1 \nabla^\beta Y_2 \nabla_\beta Y_3 + l_2(l_2+1) Y_1 \nabla_\beta Y_2 \nabla^\beta Y_3 \\
&= B(3;1,2) + \nabla^\alpha Y_1 \nabla_\alpha Y_2 Y_3 \\
&\quad - l_2(l_2+1) \nabla_\alpha Y_1 \nabla^\alpha Y_2 Y_3 - Y_1 \nabla^\beta Y_2 \nabla_\beta Y_3 + l_2(l_2+1) Y_1 \nabla_\beta Y_2 \nabla^\beta Y_3 \\
&= B(3;1,2) + (1 - l_2(l_2+1))(A(3;1,2) - A(1;2,3))
\end{aligned} \tag{18}$$

which is

$$B(1;2,3)-B(3;1,2)=(1-l_2(l_2+1))(A(3;1,2)-A(1;2,3)) \tag{19}$$

Adding (17) and (19) gives Eq.(14b).
c)

$$\begin{aligned}
D(1;2,3) &= \int \nabla_\alpha \nabla_\beta Y_1 \nabla^\alpha Y_2 \nabla^\beta Y_3 \\
&= -\int \nabla_\beta Y_1 \nabla_y^2 Y_2 \nabla^\beta Y_3 - \nabla^\beta Y_1 \nabla^\alpha Y_2 \nabla_\alpha \nabla_\beta Y_3 \\
&= l_2(l_2+1) \int \nabla_\beta Y_1 Y_2 \nabla^\beta Y_3 + Y_1 \nabla_\beta \nabla_\alpha Y_2 \nabla^\beta \nabla^\alpha Y_3 + Y_1 \nabla_\alpha Y_2 \nabla^\beta \nabla^\alpha \nabla_\beta Y_3 \\
&= l_2(l_2+1) \int \nabla_\beta Y_1 Y_2 \nabla^\beta Y_3 + Y_1 \nabla_\beta \nabla_\alpha Y_2 \nabla^\beta \nabla^\alpha Y_3 \\
&\quad + R_{\alpha\gamma} Y_1 \nabla^\alpha Y_2 \nabla^\gamma Y_3 + Y_1 \nabla_\alpha Y_2 \nabla^\alpha \nabla_y^2 Y_3 \\
&= l_2(l_2+1) A(2;3,1) + B(1;2,3) + (1 - l_3(l_3+1)) A(1;2,3)
\end{aligned} \tag{20}$$

which is equivalent to Eq.(14c).
d)

$$\begin{aligned}
G(1,2,3) &= \int \nabla_\alpha \nabla_\beta Y_1 \nabla_\beta \nabla_\gamma Y_2 \nabla^\gamma \nabla^\alpha Y_3 \\
&= -\int \nabla_\gamma \nabla_\alpha Y_1 \nabla^\alpha \nabla^\beta \nabla^\gamma Y_2 \nabla_\beta Y_3 - (1 - l_1(l_1+1)) \int \nabla_\gamma Y_1 \nabla^\beta \nabla^\gamma Y_2 \nabla_\beta Y_3 \\
&= -\int \nabla_\gamma \nabla_\alpha Y_1 (R_\alpha{}^\beta{}_\gamma{}^\delta \nabla_\delta Y_2 + \nabla^\beta \nabla_\alpha \nabla^\gamma Y_2) \nabla_\beta Y_3 + (l_1(l_1+1) - 1) \int \nabla_\gamma Y_1 \nabla^\beta \nabla^\gamma Y_2 \nabla_\beta Y_3 \\
&= (l_1(l_1+1) - 1) D(2;3,1) - \int \nabla_y^2 Y_1 \nabla_\beta Y_2 \nabla^\beta Y_3 \\
&\quad + \int \nabla^\beta \nabla^\alpha Y_1 \nabla_\alpha Y_2 \nabla_\beta Y_3 - \int \nabla_\gamma \nabla_\alpha Y_1 \nabla^\beta \nabla_\alpha \nabla^\gamma Y_2 \nabla_\beta Y_3
\end{aligned}$$

$$
\begin{aligned}
&= (l_1(l_1+1)-1)D(2;3,1) + l_1(l_1+1)A(1;2,3) + D(1;2,3) - l_3(l_3+1)B(3;1,2) \\
&\quad + \int \nabla^\beta \nabla_\gamma \nabla_\alpha Y_1 \nabla^\alpha \nabla^\gamma Y_2 \nabla_\beta Y_3 \\
&= (l_1(l_1+1)-1)D(2;3,1) + l_1(l_1+1)A(1;2,3) + D(1;2,3) - l_3(l_3+1)B(3;1,2) \\
&\quad + \int (R^\beta{}_{\alpha\gamma}{}^\delta \nabla_\delta Y_1 + \nabla_\alpha \nabla^\beta \nabla_\gamma Y_1) \nabla^\alpha \nabla^\gamma Y_2 \nabla_\beta Y_3 \\
&= l_1(l_1+1)D(2;3,1) + l_1(l_1+1)A(1;2,3) - l_3(l_3+1)B(3;1,2) \\
&\quad + l_2(l_2+1)A(2;3,1) + l_2(l_2+1)D(1;2,3)
\end{aligned}
\tag{21}
$$

which is Eq.(14d).

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